RELATIVITY

IN CURVED SPACETIME

Life Without Special Relativity

Eric Baird

Relativity *in Curved Spacetime*

Life without special relativity

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I do not see any reason to assume that … the principle of general relativity is restricted to gravitation and that the rest of physics can be dealt with separately on the basis of special relativity … I do not believe that it is justifiable to ask: what would physics look like without gravitation?

Albert Einstein, 1950

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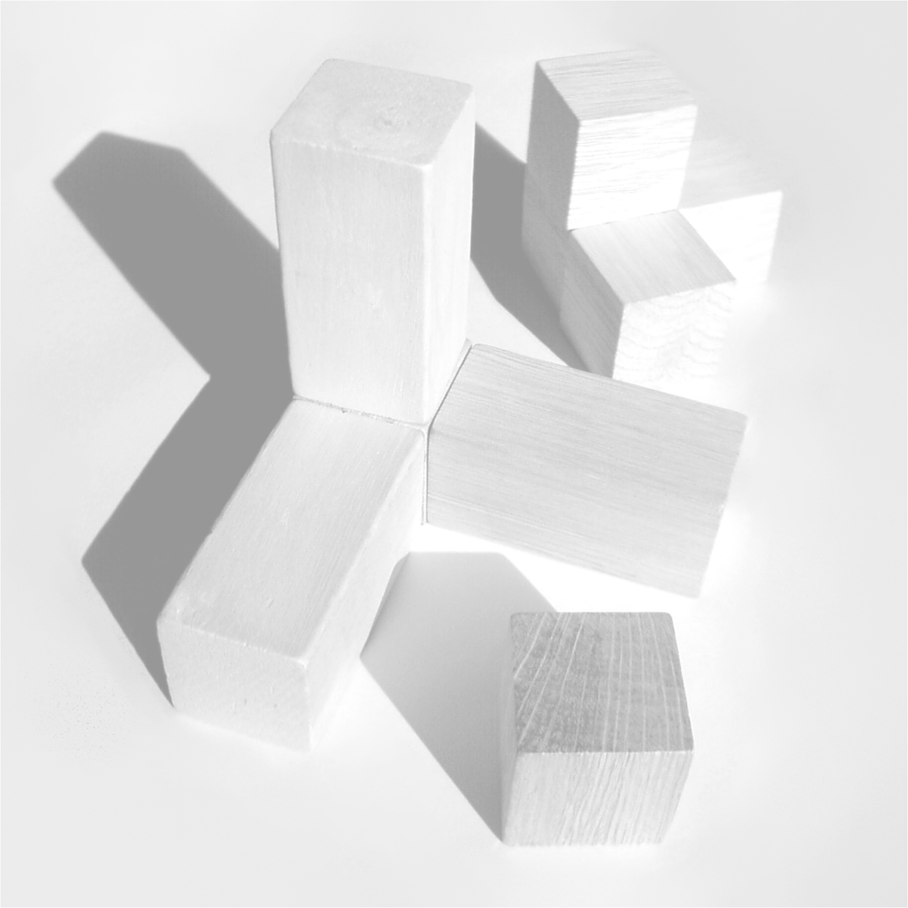
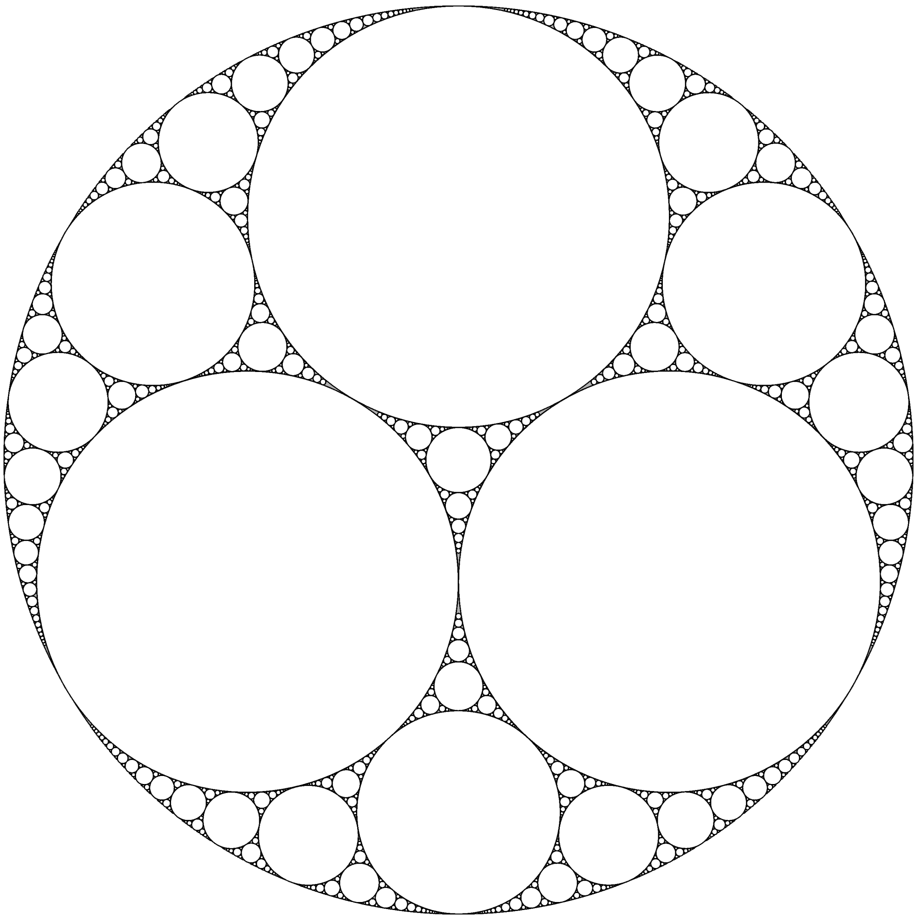
PART I

BACKGROUND PHYSICS





1
The Speed of Light

> *" … it is now certain from the phenomena of Jupiter's satellites, confirmed by the observations of different astronomers, that light is propagated in succession, and requires about seven or eight minutes to travel from the sun to the earth."*

Isaac Newton, "Principia…" (Scholium to Proposition XCVI)

> *" Where lighter gases, circumfused on high,*
> *Form the vast concave of exterior sky;*
> *With airy lens the scatter'd rays assault,*
> *And bend the twilight round the dusky vault; "*

Erasmus Darwin on atmospheric lensing, "The Botanic Garden", 1791

> *" Every child at school knows, or believes he knows, that this propagation takes place in straight lines with a velocity c= 300,000 km./sec. "*

Albert Einstein, "Relativity …"

> *" … the opposite of a profound truth may well be another profound truth. "*

Niels Bohr

> *" The principle of the constancy of the velocity of light holds good according to this theory in a different form from that which usually underlies the ordinary theory of relativity. "*

Albert Einstein, "The influence of gravitation on the propagation of light", (section 3), 1911

The Pleiades star cluster:
Its central "question mark" of young bright, blue stars spans a distance of about ten lightyears, and is thought to be approximately ~400 lightyears away (estimates vary)

1.1: Light is pretty fast

a sense of scale

The **speed of light** is approximately 300,000 kilometres per second, or a little over 186,000 miles per second in old measures. The official speed is 299,792,458 metres per second.

To get some idea of just how fast this is, the **Earth** is about 12,756 km across at the equator, and its equatorial circumference is just over 40,000 km, so a lightbeam bouncing around a series of mirrors could circle the Earth seven-and-a-bit times in one second. Or, a lightsignal created on the Earth's surface and aimed at the **Moon** would manage, in that one second, to get about four-fifths of the way there.

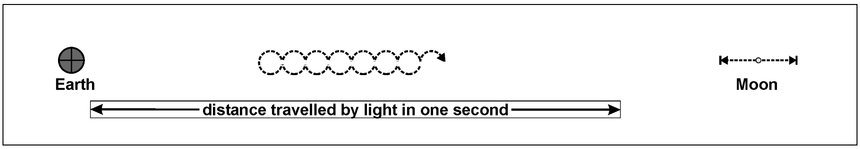


Figure 1-1: The distance covered by light in a second

Light is *so* fast that it makes a very useful ruler for describing astronomical distances, where these distances expressed in kilometres or miles would start to boggle the mind. We can say that the Earth's equator is roughly 0.135 **lightseconds** (**ls**) around, the Moon's average distance from us is about 1.2 lightseconds, the **Sun's** radius is about two-and-a-third lightseconds (so that the Moon's entire orbit would fit comfortably inside the Sun), and the distance from the Sun to the Earth is a shade over 500 lightseconds (light from the Sun takes just over eight minutes to reach us).

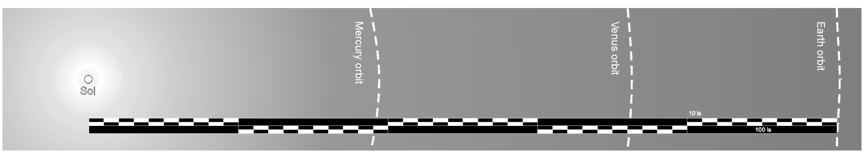


Figure 1-2: Sun→Earth, ~500 seconds

Pluto is nearly 40 times further away from the Sun, at a distance of just under 20,000 ls.

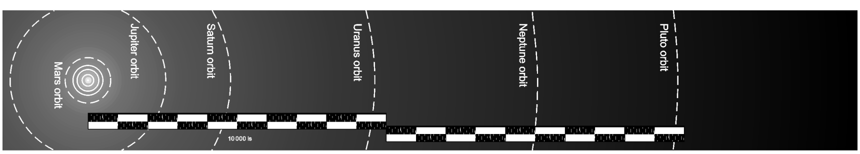


Figure 1-3: Sun→Pluto, ~20,000 seconds, approx 5½ hours

For interstellar distances the numbers get a bit big even when they're counted in lightseconds, so we use **light*years*** ("**ly**") instead, with one lightyear being, logically enough, the distance that light moves in one Earth year (nearly 32 million times bigger than our lightsecond). The nearest star to our solar system (**Alpha Centauri**) is about 4.3 ly away, and **Sirius**, the brightest star in our sky, is 8.7 ly away.

Our ten nearest stars just fit into a sphere 10 ly across.

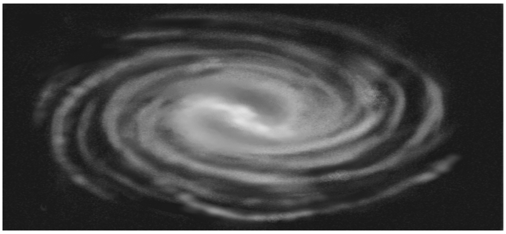


Figure 1-4: Milky Way: over 100,000 ly across

Our own galaxy is over a hundred thousand lightyears across, and the nearest galaxy of a similar size, **Andromeda** (also known as "**M31**"), is about two million lightyears away. The observable universe extends over a distance of billions of lightyears.

1.2: Lightspeed varies

We all know that the speed of light is constant, right? Well, that's not entirely true, because, in a very real physical sense, lightspeed can be said to appear to vary from place to place and from time to time, if it's provided with a good enough reason. Lightspeed "over there" can seem to be different to lightspeed "over here". However, for reasons we'll get into shortly, lightspeed "here-and-now", measured in a vacuum, always gives the same value ("lightspeed is *locally* constant"). The "local" part of that phrase is quite important.

lightspeed is affected by matter

Although light travels more slowly through regions containing matter, the speed of light in air at sea level is still quite high, with its speed only reduced by about three parts in a thousand. For light in glass or water we get down to values around two-thirds or three-quarters of the "official" speed of light, and electromagnetic signals passing down copper wires are supposed to travel at about two-thirds of standard lightspeed. These rough figures change with density: the more matter we squeeze into a region, the longer it takes light to travel through it.

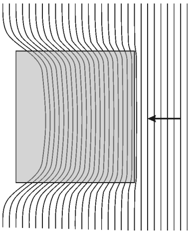


Figure 1-5: Light moves more slowly though solid objects

This trend doesn't just apply to the spaces between atoms: it also applies to the spaces between planets and stars. **Gravitational fields** affect lightspeeds, and the more stars we squeeze into a region, the longer it takes light to cross that region.

Modern gravitational theory also tells us that objects should *drag* nearby light when they accelerate or rotate. The (reduced) speed of light in a region containing particulate matter can also be *biased* by the velocity of the particles: light within a moving block of glass should find it easier to move in the same direction as the glass than in the opposite direction.

It seems that just about anything that we might do in a region can affect the speed of light. To exclude these complicating effects we say that the speed of light is only effectively constant **in a vacuum** ("*in vacuo*"), when these complicating factors aren't reckoned to be significant.

lightspeed and gravity

Gravity affects the behaviour of light. If we aim a beam of light across a region that contains a gravity-source, the signal should take longer to reach the other side than if the gravity-source wasn't there – this is known as the **Shapiro effect**.

Because of this gravitational light-slowing, if we build two identical "light-clocks" whose "ticks" are pulses of light bouncing back and forth between two parallel mirrors, and put one of these clocks into deep space, it should "tick" faster (for a given "official" distance between its plates) than a matching clock back on the laboratory bench in Earth's gravity.

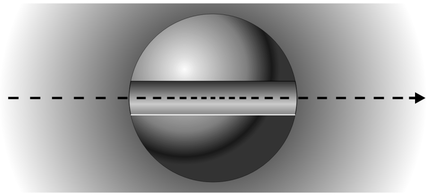


Figure 1-6: Shapiro effect : Light takes longer to cross a region where the gravitational field is stronger

This gravitational light-slowing effect doesn't depend on anyone necessarily being able to feel any sort of gravitational "pull". If we put an observer into a scooped-out hollow at the centre of the Earth, they may insist that their region *seems* to be "gravity-free", because there won't be any overall force pulling them in any particular direction. They'll feel weightless. But *we'll* still insist that their background gravitational field is stronger then ours, that their region is "downhill" from ours, and that their speed of light should be correspondingly lower.

For a particularly extreme example, we might imagine a lump of transparent material being compacted down to form a black hole. As the material compacts, light should take progressively longer to move through the region and through the lump, and once it's collapsed to less than its critical **event horizon** radius, simple calculations for how long it takes light to cross the region become *infinite*, because the light never manages to get out at all.

1.3: Lightspeed is not just the speed of light

Light is now generally understood to be an **electromagnetic wave**: the motion of an **electric field** is supposed to cause **magnetic** side-effects, and the motion of a ***magnetic*** **field** is supposed to cause ***electrical*** side-effects. The disturbances in the electrical and magnetic fields are supposed to move at the same speed – ***c*** – (for "***c***elerity", meaning "speediness") and when we create an electromagnetic disturbance or fluctuation, the resulting electromagnetic ripples scoot away from the source, hand-in-hand, as a stable electromagnetic ("**EM**") wave.

In the last half of the Nineteenth Century, **James Clerk Maxwell** calculated the speed that these hypothetical electromagnetic waves should have, and found that they had the same speed as light – light *was* an electromagnetic wave that just happened to be within the range that our eyes can see.

The spread of EM **wavelengths** that can be seen by the human eye is quite small – about an octave – but this corresponds to a range that is particularly useful. Within this range, human eyes can only properly distinguish between three overlapping colour bands centred on "**red**", "**green**" and "**blue**", and computer monitors and TV sets try to mimic the full range of visible colours by adjusting the balance of red, green and blue light for each pixel. Longer wavelengths make up the **infra-red**, **microwave** and **radio** parts of the **EM spectrum**, and shorter wavelengths are classed as **ultraviolet** ("**UV**") light, **x-rays** and **gamma rays**.

1.4: Lightspeed affects inertia

But ***c*** isn't just the speed of electromagnetism, it's the default speed at which electric and magnetic disturbances carry *information* across space – light just happens to be the handiest (and cleanest) example available. We can think of ***c*** as being the speed of *communication* without having to know anything terribly much about the properties of light or about electromagnetism in general. This "communication speed" or "reaction speed" seems to be fundamentally tied into the rate at which everyday processes occur, and affects other basic properties such as the apparent **inertial mass** of objects.

A ball's **inertia** can be defined according to how easy it is to get it to move (for instance, by hitting it with a baseball bat with a known amount of force, and seeing how quickly it leaves the bat after the impact). But when atoms in materials jostle against each other, the things that interact and bump together are the atoms' external electric fields, so the speed of light also controls the rate at which everyday forces are transmitted though solid objects.

If we reduce a region's speed of light, the forces from our bat will be (electromagnetically) transferred *to* the ball and *through* the ball's structure more slowly, and the ball will take longer to react to the impact. If the amount of energy in the moving ball is expressed as a measure of the ball's velocity compared to the speed of light, then for a fixed amount of energy, the speed of the ball leaving our bat should also be lower. Our ball seems to have an increased resistance to the applied force, and seems to have greater inertial mass.

1.5: Lightspeed controls timeflow

But there's more to it than this. Reducing ***c*** means that all collisions between molecules inside the bat and inside the batsman's body should *also* now take place at a reduced speed, and so should the chemical reactions that result from those collisions. The batter thinks more slowly because the nerve impulses travelling through their body and brain move more slowly. If they wear a mechanical wristwatch, the watch ticks at a rate determined by the time that a flywheel takes to rock from side to side under the influence of a spring, and increasing the little wheel's inertia makes it more resistant to acceleration: the watch ticks more slowly. Increasing the inertia of an *electronic* watch makes the quartz timing crystal resonate at a lower frequency, and if we happen to have an atomic clock handy, the frequencies generated by *that* clock will be slowed, too. If all this is taking place at Stagg Field, University of Chicago, above the site used by the **Manhattan Project**, increasing the inertia of any radioactive residues still under the ground will enhance their stability, and give them correspondingly longer decay times.

If a reduction in the speed of light makes *all* these different sorts of clocks run more slowly, by the same amount, then perhaps the simplest description of what is happening is that *time itself* is passing more slowly in the region of slower lightspeed.

1.6: Lightspeed is <u>locally</u> constant

But what about the local observer?

A far as our batter is concerned, *nothing seems to have changed*. Since their brain is slowed by the same rate as all the other processes around them, they're entitled to believe that the bat and ball are behaving normally. All their clocks seem to be keeping perfect time with each other. Unless they start comparing their rate of timeflow with that *outside* the pitch, or comparing gravitational field strengths, they don't have any obvious way of telling that the speed of light is any different. Even if there happens to be a University of Chicago lightspeed-testing experiment going on at the side of the pitch, bouncing light between reflectors carefully placed around the stand and measuring the time taken for the light to perform a complete circuit, the equipment won't be able to show that anything has changed.

We can argue that light *ought* to be taking longer to travel across the stadium, but since the experiment's local reference clocks should be taking longer between ticks by the same proportion, the equipment should be obstinately displaying the same numerical value for the speed of light that it showed before we made our change.

This is what we mean when we say that lightspeed is **globally variable but locally constant**: an outsider beaming light across the time-slowed region would be able to tell that it was taking longer than normal to reach the other side, but every observer making strictly ***local*** observations of the speed of light in a vacuum ought to always get exactly the same figure. Any uniform gravitational effects that modify *local lightspeeds* also modify the rest of the ***local physics*** in such a way as to make the change invisible to any *locally-calibrated* detection equipment. At least, that's the theory.

1.7: Lightspeed is now defined as constant

The speed of light in highly controlled situations can now be measured to exquisite accuracy, and these measurements are exquisitely repeatable. As the accuracy with which we were able to measure the speed of light started to approach the accuracy with which we could define distances and times, we needed a new approach. The new atomic clocks gave us a better reference "time-base", but we needed an equally accurate and replicable measure of distance.

In the end we took a pragmatic approach: we now *define* lengths by the distance that light travels in a known time, and the metre is now *defined* as the distance that light moves in exactly 1/ 299 792 458 of a second.

1.8: The gravity well

If we follow these definitions and use light-signals to measure distances, we can end up saying that, in effect, a gravitational field's light-slowing abilities seem to modify *the amount of space that we measure* inside a region:

Let's take a cube-shaped box and a stopwatch and measure how long it takes light to travel from one side of the box to the other. If we now place a microscopic black hole at the centre of the box and repeat the exercise, rays that pass closer to the box centre should take longer to reach the other side. Using these light rays to map out distances, we can then say that the middle of the box seems to "contain more distance" that we'd expect from its external dimensions: it seems to be bigger on the inside than the outside.

We can express this by drawing an **embedding diagram** to express these modified light-distances. If we take a cross-section through the box when it's empty and map out its insides, we might end up with a map something like this:

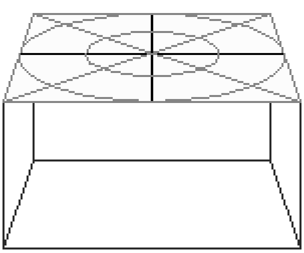


Figure 1-7: Cross section through an empty box: flat spacetime

This is what we'd normally refer to as an example of **flat geometry**.

Now we'll repeat the exercise when the box contains its tiny central gravity-source.

In the new map, the lightbeams will take progressively longer to cross the region depending on how close they get to the centre of the box, so the new map will describe space as seeming to contain more "space per unit volume" towards its middle, with the "spatial density" appearing to be higher towards the middle of the box. We might use darker shading and/or contour lines to describe the apparent change in density, like this:

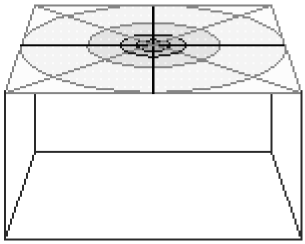


Figure 1-8: Contour map of apparent spatial density around a gravity-source

Finally, we might decide that we don't like this approach of using contour lines and shadings: we'd prefer the map's distances to be an accurate "scale" representation of the light-distances involved. The problem with this is approach that the extra distances in the centre of the box won't fit politely into a "flat" map – in order to cram them in, we have to make our map bulge in the middle: we have to **extrude** our two-dimensional map of the slice through the box into a third dimension, in order to get everything to fit in.

This gives us a sort of curved "funnel" shape, something like this:

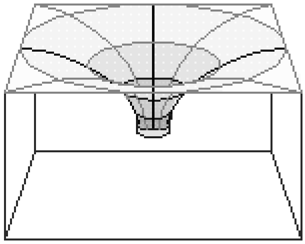


Figure 1-9: Gravity-well

We refer to this distinctive shape as a **gravitational well**. We usually see it drawn using a horizontal plane and with the well's throat pointing "down", but that's mostly because the diagram looks more natural to us that way. We could equally well have drawn the diagram with the funnel bulge pointing upwards, like an odd-looking mountain – what's important here is the way that distances are packed *inside* the shape, and the resulting angles.

Shapes drawn on this curved surface no longer obey the ancient rules of conventional classical Greek geometry as described by **Euclid**, and we refer to geometry carried out in these sorts of curved surfaces (or in curved spaces) as **non-Euclidean geometry**. The rules concerning non-Euclidean surfaces and volumes weren't properly worked out until the middle of the Nineteenth Century, by pioneers **Karl Friedrich Gauss** and **Georg Friedrich Riemann**, and we'll be dealing with some of the implications of this curvature in section 3.

1.9: Light travels in straight lines. Except when it doesn't

Another thing that we learn in school is that light travels in straight lines. Except that, of course, we know that it often doesn't. Lightrays bend when they move from air to water or glass to air, and light also *seems* to bend even when it's only skimming the surfaces of objects.

A transparent body designed with a special shape to exploit this first effect and make lightrays converge or diverge in a particular way is called a **lens**, and "lensing" effects are quite common: if we look above a hot radiator in winter, objects behind it seem to "ripple" as the warmer and less dense air rising from the radiator creates variations in the way that light moves through the region, and similar lightspeed-lensing effects in air above hot sand are supposed to be to blame for the phenomena of desert **mirages**.

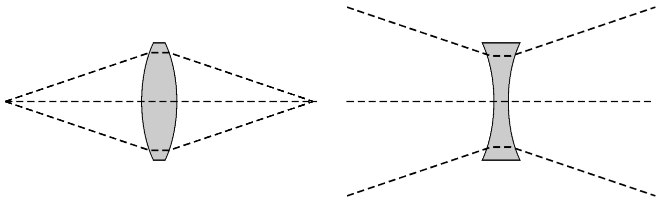


Figure 1-10: Conventional lensing effects

Newton and his contemporaries recognised that these deflections could be explained by variations in the speed that light as it encountered materials with different optical properties (section 5), and Newton also went on to explore how conventional gravitational effects might be explained by similar variations in lightspeed, caused by the regional variations in density of a gravitational medium.

If conventional lenses deflect light by changing lightspeeds in a region, and gravitational fields also affect lightspeeds, does *gravity* deflect lightbeams, too?

The short answer is: yes. Suppose that we repeat our "box" experiment and rig things so that one face of the box illuminates all-at-once. In the case of the empty box, we expect the resulting electromagnetic shockwave to move away from the box wall with a nice flat leading edge and travel as a **plane wave** to the opposite side of the box, hitting it face-on. Because, really, it has no obvious excuse to do anything else. We can then draw in lines at right angles to this plane wave, and describe how these **rays** describe the paths that light moves along when travelling from one side of the box to the other. They are nice straight lines.

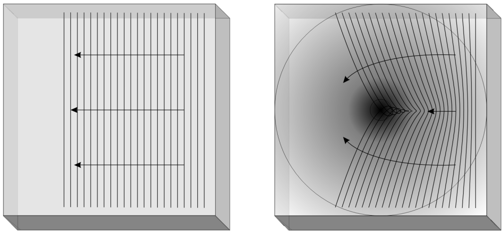


Figure 1-11: Gravitational deflection of light

But things change when we put our tiny gravity-source back into the box. The rays moving through the centre of the box now take longer to reach the far end than the rays further away from the gravity-source, so the centre of our wavefront progresses more slowly than its edges, and the result of "holding back" the middle of the wave is that the wavefront ends up distorted, and parts of its surface begin to tilt more towards the direction of the gravity-source.

We refer to the deflection of lightbeams around a gravity-source as **gravitational lensing**, and astronomers have collected some nice examples of images that seem to show this effect in action. When light bends around the gravitational field of an extremely massive, distant object (such as a galaxy) it produces quite a distinctive-looking effect.

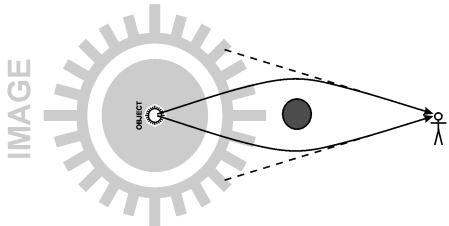


Figure 1-12: Gravitational lensing

These images tend to be referred to as **Einstein rings** (although since **Orest Chwolson** suggested the effect earlier, it might be more proper to refer to them as **Chwolson rings**).

Figure 1-13: Results of "perfect" and "irregular" gravitational lensing

Although we've only recently had the technology required to *see* the effects, the *idea* of gravitational lensing isn't especially novel, and when Einstein submitted his paper on the subject, he apologised to the editor for sending in something so unimportant (explaining that he'd only written it to stop someone hassling him). The general problem – of calculating lensing effects caused by a medium whose density changes with distance – is quite old, and Newton is known to have carried out similar calculations for the more conventional lensing effects caused by a planet's atmosphere (which become weaker with distance in a broadly similar way). Gravitational lensing would have been a natural result of Newton's physics.

1.10: Light used to define a straight line

The gravitationally-deflected rays that we've drawn look suspiciously like the sort of lines that we'd end up with it we tried to draw a set of parallel straight lines across our gravity-well shape in Figure 1-9 using a flexible ruler, without realising that the surface was curved. The curvature could fool us into drawing lines that followed the curve of the surface ("**geodesics**") so that they ended up crossing each other at strange angles, and this suggested to some clever mathematicians in the Nineteenth Century that perhaps our innocent-looking curved-space map might have more powerful properties: it might describe and map *all of* the properties of light in a gravitationally-distorted region, and might be able to generate a full *geometrical* description of how gravitational fields influenced nearby matter.

The idea of modelling gravitational physics as the result of **spatial curvature** was brilliant, but the mathematicians involved couldn't seem to get it work. Some vital component was still missing. But before we can look at what it was, we need to look again at how gravity interferes with some of our older ideas about light and matter.

2
Gravity, Energy and Mass

> *" Are not gross Bodies and Light convertible into one another, and may not Bodies receive much of their Activity from the Particles of Light which enter their Composition? For all fix'd Bodies being heated emit Light so long as they continue sufficiently hot, and Light mutually stops in Bodies as often as its Rays strike upon their Parts, as we shew'd above. ...*
>
> *The changing of Bodies into Light, and Light into Bodies, is very conformable to the Course of Nature, which seems delighted with Transmutations. ... And among such various and strange Transmutations, why may not Nature change Bodies into Light, and Light into Bodies? "*

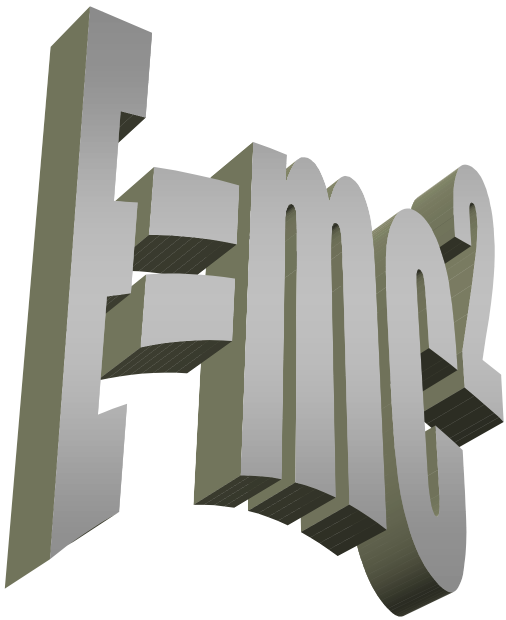
Isaac Newton, "Opticks" 3:1:Qu30:

> *" If the theory corresponds to the facts, radiation conveys inertia between the emitting and absorbing bodies. "*

Albert Einstein,
"Does the inertia of a body depend upon its energy-content?", 1905

> *" 'And we know now that the atom, that once we thought hard and impenetrable, and indivisible and final and – lifeless, is really a reservoir of immense energy. … A little while ago we thought of the atoms as we thought of bricks, as solid building material, as substantial matter, as unit masses of lifeless stuff, and behold! these bricks are boxes, treasure boxes, boxes full of the intensest force. '*
>
> *' Why does not all the uranium change to radium and all the radium change to the next lowest thing at once? Why this decay by driblets; why not a decay en masse? ... Suppose presently we find it is possible to quicken that decay?'*
>
> *' We cannot pick that lock at present, but – … – we will. ' "*

H.G. Wells, "The World Set Free", 1915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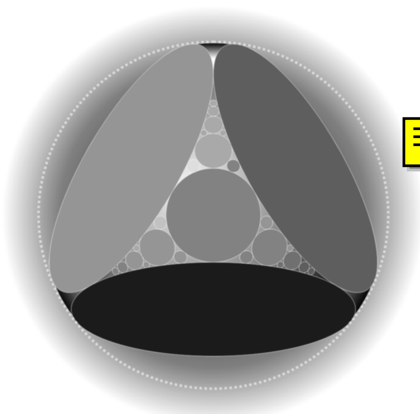
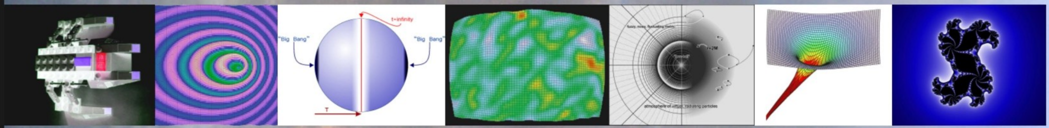


RELATIVITY IN CURVED SPACETIME

Eric Baird

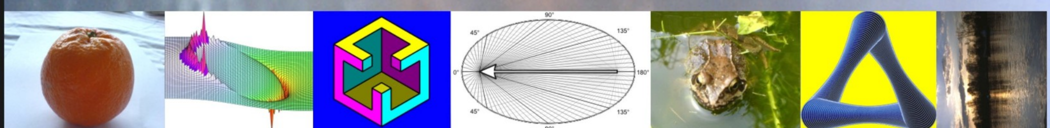
Life Without Special Relativity

> " I do not see any reason to assume that ... the principle of general relativity is restricted to gravitation and that the rest of physics can be dealt with separately on the basis of special relativity ... I do not think that such an attitude, although historically understandable, can be objectively justified. ... In other words, I do not believe that it is justifiable to ask: what would physics look like without gravitation? "
>
> Albert Einstein, "Scientific American", April 1950



Throughout the Twentieth Century, the word "relativity" has been deeply associated with Einstein's 1905 flat-spacetime "special" theory. "Special relativity" assumes the absence of matter and gravitation, and has counterintuitive consequences that have intimidated generations of students. But what if relativity theory requires spacetime to be *distorted* in order to work properly, and if special relativity (with its confusing paradoxes and pseudoparadoxes) was only ever a faulty implementation of the idea? What if a *fully* general theory of relativity should not reduce to Einstein's original 1905 theory *on principle*?

"Relativity..." takes us on a whistlestop tour of Twentieth Century physics – from black holes, quantum mechanics, wormholes and the Big Bang to the workings of the human mind, and asks: what would physics look like without special relativity?



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